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CONTROLLING CHAOTIC MOTIONS IN A TWO-DIMENSIONAL AIRFOIL USING TIME-DELAYED FEEDBACK

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In this paper, the chaotic motions of a two-dimensional airfoil are controlled by the application of the time-delayed continuous feedback method of Pyragas. The airfoil has cubic pitching stiffness and linear viscous damping when kept in an incompressible fluid flow. Four control strategies are implemented with plunging displacement, plunging velocity, pitching angle, and pitching velocity as the feedback signals. The control signal is applied to perturb the airspeed parameter. The response of the system under these four controls is compared. It is found that the feedback control signal derived from the pitching variables was found to be more effective in controlling the chaotic motion of the airfoil.

1. INTRODUCTION

It is well known that the panel flutter is one of the classical problems of self-excited systems in aeroelasticity. Flutter is a dangerous phenomenon which can cause structural failure. Dowell [1, 2], in his study of chaotic motion of panel flutter has shown that for sufficiently large in-plane load and moderate to large V, where V is the supersonic airspeed, chaotic motions occur. The chaotic motion seems to arise as a result of the presence of flow velocity and mechanical in-plane load, which govern two distinct types of instability, namely, flutter (Hopf bifurcation) and Euler buckling (sometimes called static bifurcation). Yang and Zhao [3] investigated experimentally the limit cycle flutter of an airfoil in incompressible flow with non-linear pitching stiffness. They have shown the existence of double-limit-cycle flutter and two unstable limit cycles, using the wing model with free play in pitch. Kim and Lee [4] investigated a flexible airfoil with freeplay non-linearity in pitch in the subsonic flow range. They have observed that the limit cycle oscillation and chaotic motion are highly influenced by the pitch-to-plunge frequency ratio. Price and Keleris [5] analyzed the aeroelastic response of a NACA 0012 airfoil with freedom to move in pitch only, and forced to oscillate through dynamic stall in subsonic flow. They have demonstrated that the aerodynamic non-linearities associated with dynamic stall are sufficient to cause a chaotic response. Price *et al.* [6] also considered a two-dimensional airfoil subject to incompressible flow with a structural free-play non-linearity in pitch. They have shown that the chaotic motion can exist for a single structural non-linearity in the pitch motion. Laurenson and Trn [7] analyzed a missile control surface containing free-play structural non-linearity exposed to subsonic flow. They have shown that for increasing dynamic pressure, or flight velocity, the system becomes unstable and the response tends to become divergent in nature.

It has been observed that above some critical dynamic pressure, the system exhibits divergent flutter motion. Zhao and Yang [8] investigated the chaotic motions of a two-dimensional airfoil with cubic pitching stiffness in incompressible flow. It has been shown that with steady airforce, an airfoil in incompressible flow will exhibit chaotic motions when the airspeed is higher than the linear divergent speed. Raghothama and Narayanan [9] has considered periodic and chaotic motions of a two-dimensional airfoil using the incremental harmonic balance method and studied the bifurcations by a parametric continuation technique.

In recent years, research in the area of non-linear dynamical systems has been focussed towards control of chaos [10–12]. Chaos control algorithms can be classified into feedback and non-feedback methods. The non-feedback methods suppress chaotic motion by converting the system dynamics to a periodic orbit by applying weak periodic perturbations on some control parameters or variables. The feedback methods control chaos by stabilizing a desired unstable periodic orbit (UPO) embedded in a chaotic attractor. The important feedback chaos control methods are the Ott–Grebogi–Yorke (OGY) method [10] and the continuous time-delayed feedback method of Pyragas [12]. A chaotic attractor has embedded within it an infinite number of UPOs. The OGY control method is essentially based on feedback control that stabilizes the system on a UPO. Pyragas [12] used continuous time-delayed feedback where a system parameter is perturbed in proportion to the difference between the delayed output signal and the current signal of the dynamical system.

In this paper, the chaotic motions of a two-dimensional airfoil are controlled by the application of delayed continuous feedback [12]. The airfoil has cubic pitching stiffness and linear viscous damping when kept in an incompressible fluid flow. Four control strategies are implemented with (i) plunging displacement, (ii) plunging velocity, (iii) pitching angle, and (iv) pitching velocity as the feedback signals. The control signal is applied to perturb the airspeed parameter. The response of the system under these four controls is compared.

2. DETERMINATION OF CONTROL GAIN AND DELAY TIME

The time-delayed feedback control signal of Pyragas is of the form

$$\delta p = K(x_i(t-\delta) - x_i(t)) = KD(t), \tag{1}$$

where K is the control gain, $D(t) = (x_i(t-\delta) - x_i(t))$, x is a state-space vector, p is the accessible parameter of the system, δp is the control signal which is proportional to the difference between the delayed output signal $x_i(t-\delta)$ and the current output signal $x_i(t)$, x_i is a dynamical variable of the system which is available for measurement and δ is the delay time. By calculating the dispersion $\langle D^2(t)\rangle$ for various values of δ a sequence of resonance curves, the minima of which give the values of δ for UPO stabilization is obtained. For a given δ, the dispersion $\langle D^2(t)\rangle$ is obtained for different values of control gain K. This gives a finite interval of K with minimum dispersion for which UPO stabilization may be possible. Figure 1 shows a schematic of the delayed feedback control.

3. MATHEMATICAL MODEL

Figure 2 shows a two-dimensional airfoil. The equations of motion of the airfoil with two degrees of freedom in pitch and plunge are given by Yang and Zhao [3]

$$m\ddot{h}_p + S_\alpha \ddot{\alpha} + K_h h = Q_h, \quad S_\alpha \ddot{h}_p + I_\alpha \ddot{\alpha} + M(\alpha) = Q_\alpha, \tag{2}$$

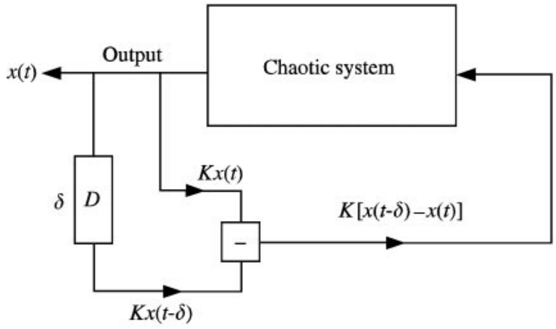

Figure 1. Schematic of delayed-feedback control system.

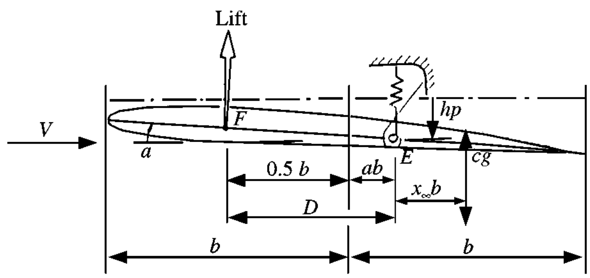

Figure 2. Sketch of the two-dimensional airfoil.

where h_p is the plunge displacement, α is the pitching angle, the number of overdots represents the order of derivative with respect to τ, the time, m is the total mass per unit span, S_α is the mass static moment, I_α is the mass moment of inertia, K_h is the plunging stiffness coefficient and $M(\alpha)$ is the non-linear stiffness term.

The unsteady aerodynamic force (Q_h) and moment (Q_α) are expressed in terms of the Theodorsen functions when the motion is simple harmonic as in the linear critical flutter case [3];

$$Q_h = -\pi\rho b^2(V\dot{\alpha} + \ddot{h} - ab\ddot{\alpha}) - 2\pi\rho VbC(k)[V\alpha + \dot{h} + (0{\cdot}5 - a)b\dot{\alpha}],$$

$$Q_\alpha = \pi\rho b^2[ab(V\dot{\alpha} + \ddot{h} - ab\ddot{\alpha}) - 0{\cdot}5Vb\dot{\alpha} - b^2\ddot{\alpha}/8] \tag{3}$$

$$+ 2\pi\rho Vb^2(0{\cdot}5 + a)C(k)[V\alpha + \dot{h} + (0{\cdot}5 - a)b\dot{\alpha}].$$

Here ρ is the air density, V is the air speed, b is the half-chord length of the airfoil, ab is the streamwise distance of the pitch axis E from the mid-chord point, $C(k)$ is the Theodorsen function for unsteady aerodynamics and k is the reduced frequency.

Considering steady airforce, and introducing the non-dimensional parameters $\mu = m/(\pi\rho b^2)$, $x_\alpha = S_\alpha/(mb)$, $r_\alpha^2 = I_\alpha/(mb^2)$, $\omega_h^2 = K_h/m$, $\omega_\alpha^2 = K_\alpha/I_\alpha$ the equations of motion become

$$\mu(\mathrm{d}^2h/\mathrm{d}t^2) + \mu x_\alpha(\mathrm{d}^2\alpha/\mathrm{d}t^2) + \mu(\omega_h/\omega_\alpha)^2h = -2Q\alpha, \tag{4}$$

$$\mu x_\alpha(\mathrm{d}^2h/\mathrm{d}t^2) + \mu r_\alpha^2(\mathrm{d}^2\alpha/\mathrm{d}t^2) + \mu r_\alpha^2\alpha = (1 + 2a)Q\alpha, \tag{5}$$

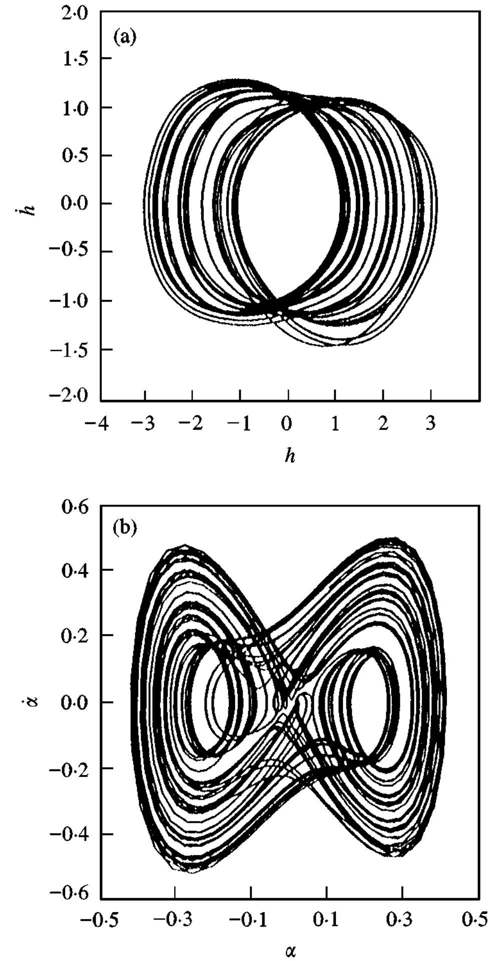

Figure 3. Phase plots depicting the chaotic motion of the airfoil for parameters $Q = 15{\cdot}6$, $e = 20$ and $d = 0{\cdot}07$: (a) plunging rate versus plunging displacement; (b) pitching angle versus pitching rate.

where $h = h_p/b$, $Q = V/(b\omega\alpha)$, $t = \omega_\alpha \tau$ is the non-dimensional time. The following parameters [8] are considered for the present study: $\mu = 20$, $a = -0{\cdot}1$, $b = 1{\cdot}0$ m, $x_\alpha = 0{\cdot}25$, $r_\alpha^2 = 0{\cdot}5$, $(\omega_h/\omega_\alpha)^2 = 0{\cdot}2$, $\omega_h = 28{\cdot}1$ Hz and $\omega_\alpha = 62{\cdot}8$ Hz.

Upon introducing the viscous damping terms and the cubic pitching stiffness term, the governing equations are obtained as

$$\ddot{h} + 0{\cdot}25\ddot{\alpha} + 0{\cdot}1\dot{h} + 0{\cdot}2h + 0{\cdot}1Q\alpha = 0, \tag{6}$$

$$0{\cdot}25\ddot{h} + 0{\cdot}5\ddot{\alpha} + 0{\cdot}1\dot{\alpha} + 0{\cdot}5\alpha + e\alpha^3 - 0{\cdot}04\,Q\alpha = 0, \tag{7}$$

where e is the non-linear stiffness factor, and the superscript dot denotes d/dt.

The parameters considered for the analysis are $e = 20$, $Q = 15{\cdot}6$, $d = 0{\cdot}07$ with one definite set of initial conditions: $(h_0, \dot{h}_0, \alpha_0, \dot{\alpha}_0) = (0, 0, 0{\cdot}01, 0)$ for which a chaotic solution exists. Figures 3(a) and (b) depict the chaotic motion of the airfoil for the above parameters and initial conditions in both plunge and pitch modes. The fourth order Runge–Kutta method has been used for numerical integration with a time step of 0·1.

4. CONTROL IMPLEMENTATION

The control signal is proportional to the difference between the output of the system at the current instant and the output of the system at an earlier time. Here $Q = Q_0 + F(t)$, where $Q_0 = 15{\cdot}6$ $(F(t) = 0)$ is the nominal value for which the motion of the airfoil is chaotic and $F(t)$ is the control perturbation given by

$$F(t) = K(x(t-\delta) - x(t)) = KD(t), \tag{8}$$

where K is an adjustable weight of the perturbation, δ is the delay time, $x(t)$ is the current output signal, $x(t-\delta)$ is the delayed output signal and $D(t) = (x(t-\delta) - x(t))$. When δ coincides with the period of the ith unstable periodic orbit (UPO), $\delta = T'_1$, then the perturbation becomes zero corresponding to this UPO. Therefore, the perturbation does not change the form of the UPO but stabilizes it.

In this paper, for the control of chaotic motions of the airfoil, four strategies are considered by measuring (i) plunging displacement, (ii) plunging rate, (iii) pitching angle, and (iv) pitching rate individually in each case as the feedback signal.

The control signal is fed back to modify the system parameter Q, which is proportional to the airspeed. The control parameters, namely, the weight K and the delay time δ are chosen appropriately to achieve the stabilization of the desired periodic motion. In experimental situations, this is done by adjusting these parameters till the amplitude of the feedback signal becomes extremely small which happens when the system moves along its UPO. For a non-autonomous system the delay time can be taken as the excitation period or multiples of it depending on the desired periodic motion. In numerical experiments, the parameter δ is determined by calculating the dispersion $\langle D^2(t)\rangle$ of the perturbation, excluding the transient process, for each value of δ. The very deep minima of the resulting figure representing the sequence of resonance curves give the value of the delay time coinciding with the periods of the UPO $\delta = T_i$. Similarly, the range of weight K is determined, for a given value of δ, by calculating the dispersion of the perturbation. The dependence of the dispersion of perturbation on the delay time, and the dependence of the dispersion of the perturbation on the weight K for a given value of δ are shown in Figures 4(a), (b), 6(a), (b), 7(a), (b) and 9(a), (b), respectively, for the four cases considered respectively.

5. RESULTS AND DISCUSSIONS

Case I: *When* $x(t) = h(t)$. Figure 4(a) shows the dependence of the dispersion on the delay time. The first minimum corresponds to $\delta = 10{\cdot}75$ and the second minimum occurs at $\delta = 21{\cdot}5$. Figure 4(b) shows the dependence of the dispersion on the control weight K for two values of δ. The values of K, for a particular value of δ, can be chosen within the interval where the dispersion is low.

The reference time period in all the four cases is taken as the non-dimensional time $t = 10{\cdot}75$. The final periodic response of the airfoil in pitch and plunge is measured with respect to this time period.

For $K = 1{\cdot}05$ and $\delta = 10{\cdot}75$, Figures 4(c)–(f) show the chaotic motion of the airfoil stabilized to a period-1 motion in plunge and period-2 motion in pitch with a very short transient after the control is activated at $t = 2000$. The control perturbation is less than 10% of the nominal airspeed value as shown in Figure 4(g). The control perturbation is very gradual and smooth with the stabilization achieved within a very short time.

For $K = 1{\cdot}05$ and $\delta = 21{\cdot}5$, the motion in plunge is stabilized to a period-1 orbit and pitch motion is stabilized to a period-2 motion as can be seen from the time histories shown

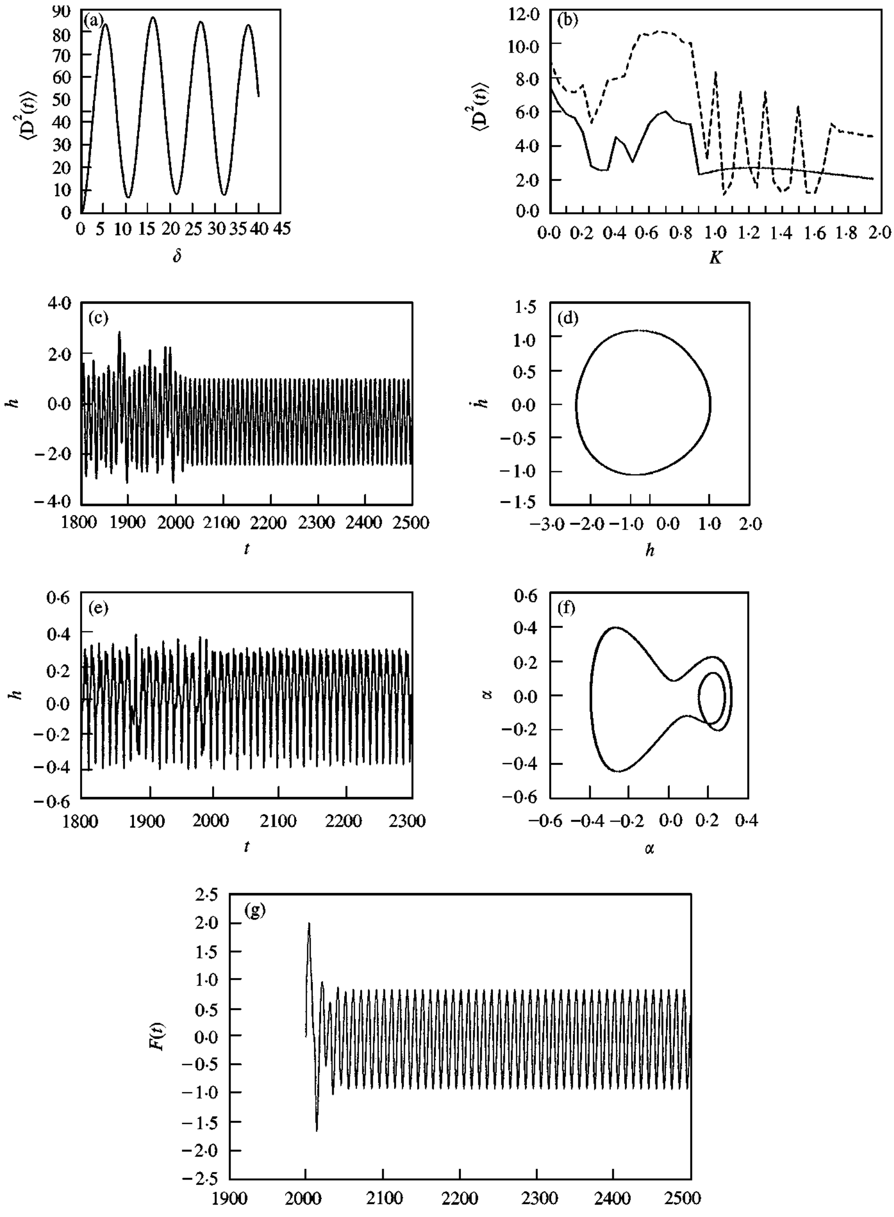

Figure 4. For case 1, when $x(t) = h(t)$: (a) dependence of the dispersion of perturbation on delay time; (b) dependence of the dispersion of the perturbation on K for two values of the delay time δ; ——, $\delta = 10{\cdot}75$; - - - -, $\delta = 21{\cdot}5$; (c) stabilization of plunging displacement ($K = 1{\cdot}05$, $\delta = 10{\cdot}75$); (d) corresponding phase plot of (c) after control transients (stabilization of period-l orbit); (e) stabilization of pitching angle ($K = 1{\cdot}05$, $\delta = 10{\cdot}75$); (f) corresponding phase plot of (e) after control transients (stabilization of period-2 orbit); (g) control perturbations ($K = 1{\cdot}05$, $\delta = 10{\cdot}75$, control activated at $t = 2000$).

in Figures 5(a) and (c) and the phase plots shown in Figures 5(b) and 5(d). Here the post-control transient is slightly larger than the earlier case with $\delta = 10{\cdot}75$. The initial control perturbation is higher.

Case II: *When* $x(t) = \dot{h}(t)$. There was no control up to the value of $K = 1{\cdot}0$ for both $\delta = 10{\cdot}75$ and $21{\cdot}5$. But for a higher value of K the plunging motion was found to be

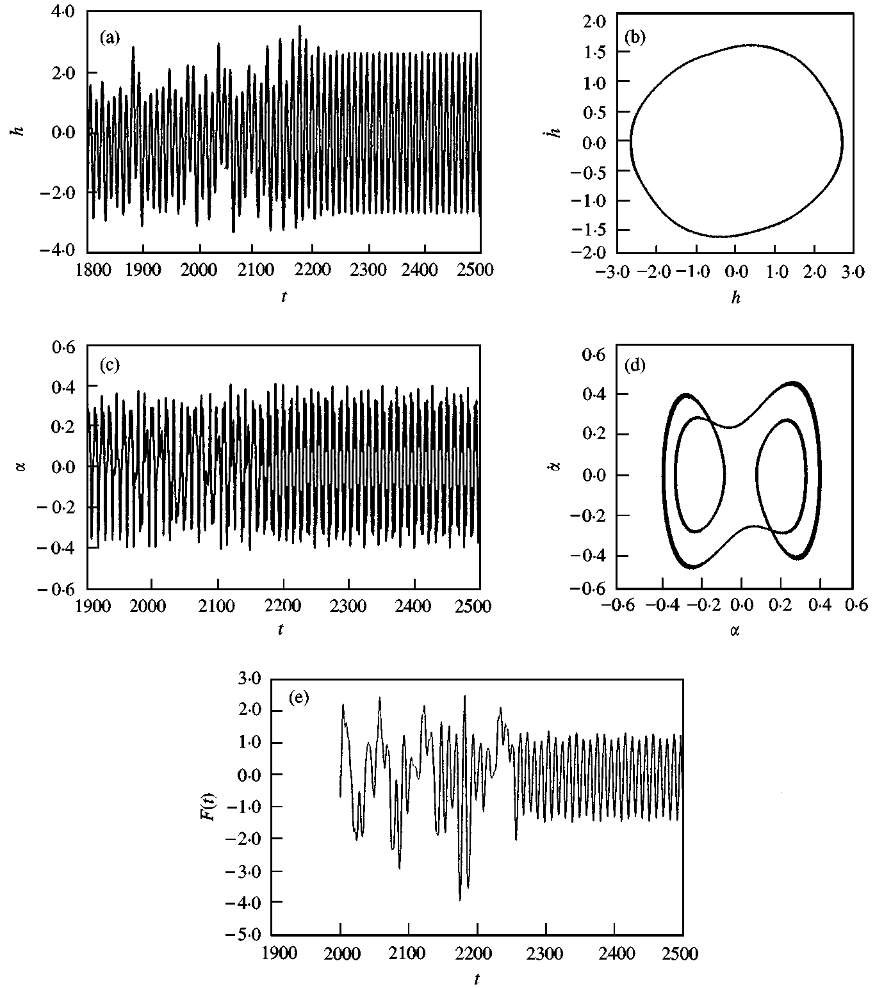

Figure 5. (a) Stabilization of plunging displacement ($K = 1{\cdot}05$, $\delta = 21{\cdot}5$); (b) corresponding phase plot of (a) after control transients (stabilization of period-1 orbit); (c) time history of pitching angle before and after control (stabilized to a period-2 oscillation); (d) corresponding phase plot of (c); (e) control perturbations ($K = 1{\cdot}05$, $\delta = 21{\cdot}5$, control activated at $t = 2000$).

stabilized but required large control perturbations. The control is not very effective at lower values of K. This may be due to the relatively high unstable values of the plunge rate. Figures 6(c)–(f) show the failure of control for $K = 0{\cdot}8$ with $\delta = 10{\cdot}75$. Figure 6(g) shows the control perturbations applied. The control is activated at $t = 2000$.

Case III: *When* $x(t) = \alpha(t)$. For $K = 0{\cdot}3$ and $\delta = 10{\cdot}75$, Figures 7(c)–7(f) show the time histories and phase plots of the plunging and pitching motion stabilized to a period-2 motion. The control perturbations required are very small as shown in Figure 7(g). The transient time after control is longer. The interval of K over which the control is very effective is very small as seen in Figure 7(b).

For $K = 0{\cdot}3$ and $\delta = 21{\cdot}5$, Figures 8(a)–(d) show the motion in plunging and pitching again describing a period-2 motion. The control perturbations are very small as shown in Figure 8(e). In this case, the control transients are shorter than when the delay time was $\delta = 10{\cdot}75$.

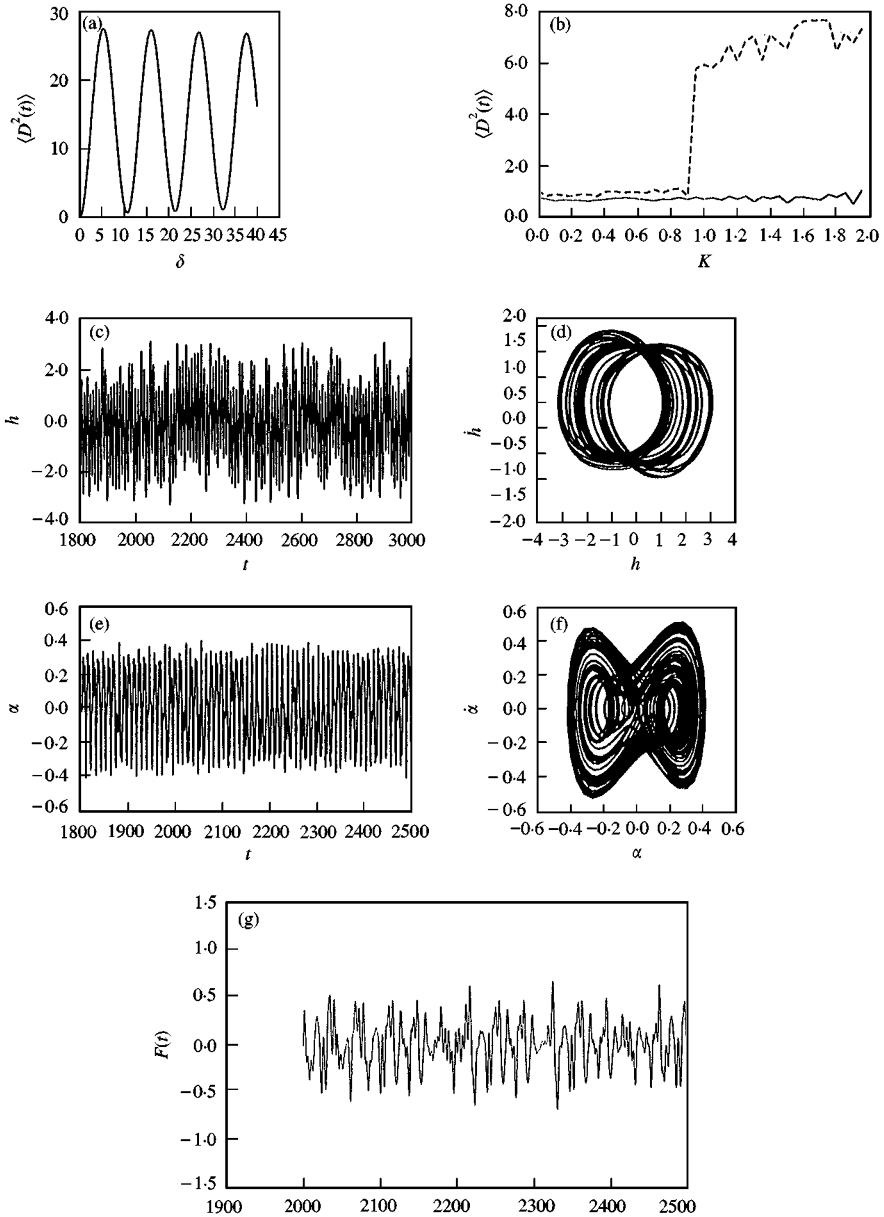

Figure 6. For case II, when $x(t) = \dot{h}(t)$: (a) dependence of the dispersion of perturbation on delay time; (b) dependence of the dispersion of the perturbation on K for two values of the delay time δ; ——, $\delta = 10{\cdot}75$; - - - -, $\delta = 21{\cdot}5$; (c) time history of plunging displacement before and after control ($K = 0{\cdot}8$, $\delta = 10{\cdot}75$); (d) corresponding phase plot of (c) after control transients (chaotic); (e) time history of pitching angle before and after control ($K = 0{\cdot}8$, $\delta = 10{\cdot}75$); (f) corresponding phase plot of (e) after control transients (chaotic); (g) control perturbations ($K = 0{\cdot}8$, $\delta = 10{\cdot}75$, control activated at $t = 2000$).

Case IV: *When* $x(t) = \dot{\alpha}(t)$. For $K = 0{\cdot}4$ and $\delta = 10{\cdot}75$, Figures 9(a)–(f) show the time histories and phase plots of the motion in plunging and pitching controlled to a period-1 and -2 cycle respectively. The control perturbations needed are very small as shown in Figure 9(g). The control perturbation is smooth and gradual. The control transients are shorter.

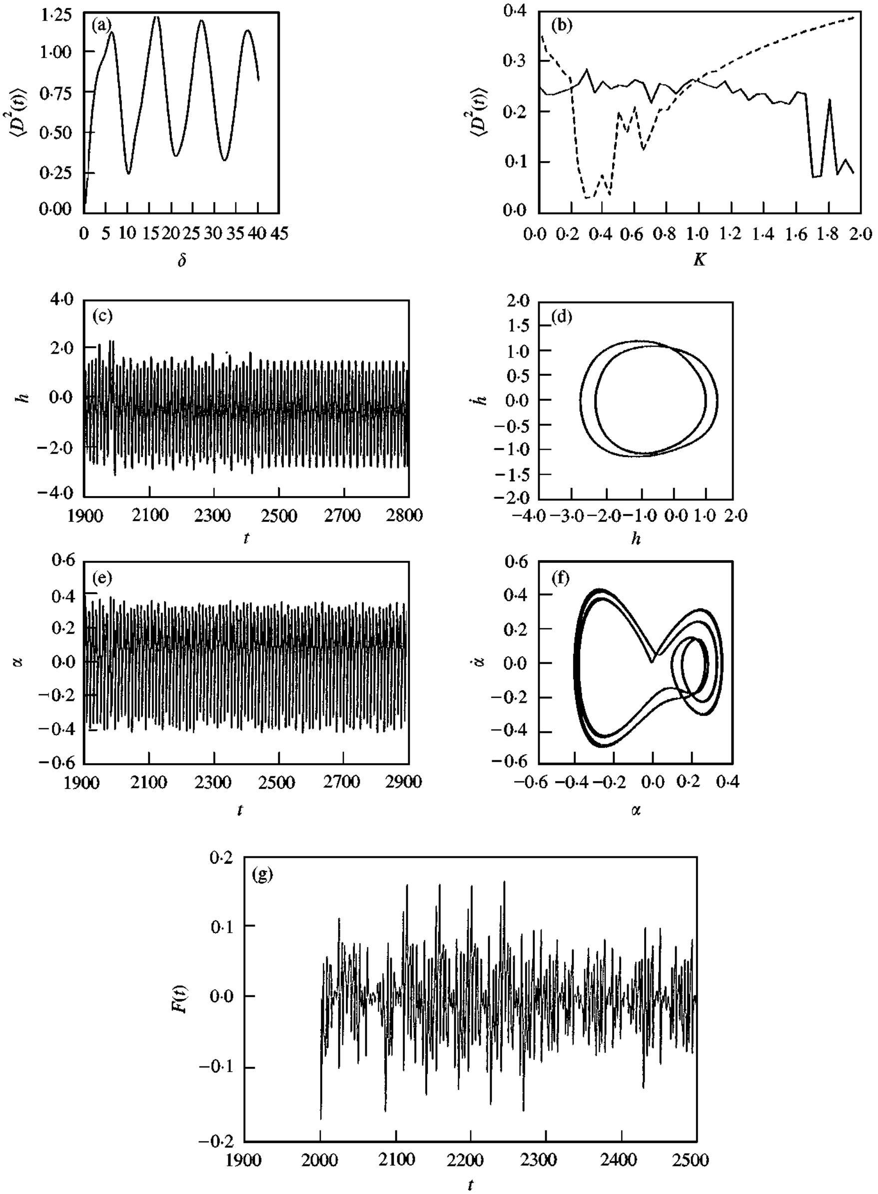

Figure 7. For case III, when $x(t) = \alpha(t)$: (a) dependence of the dispersion of perturbation on delay time; (b) dependence of the dispersion of the perturbation on K for two values of the delay time δ; ——, $\delta = 10{\cdot}75$; - - - -, $\delta = 21{\cdot}5$; (c) stabilization of plunging displacement ($K = 0{\cdot}3$, $\delta = 10{\cdot}75$); (d) corresponding phase plot of (c) after control transients (stabilization of period-2 cycle); (e) stabilization of pitching angle ($K = 0{\cdot}3$, $\delta = 10{\cdot}75$); (f) corresponding phase plot of (e) after control transients (period-2 cycle); (g) control perturbations ($K = 0{\cdot}3$, $\delta = 10{\cdot}75$, control activated at $t = 2000$).

For $K = 0{\cdot}4$ and $\delta = 21{\cdot}5$, Figures 10(a)–(d) show the plunging and pitching motion stabilized into a period-1 and -2 cycle respectively. The control perturbations required are very small and less than 5% of the nominal value of the airspeed parameter as shown in Figure 10(e).

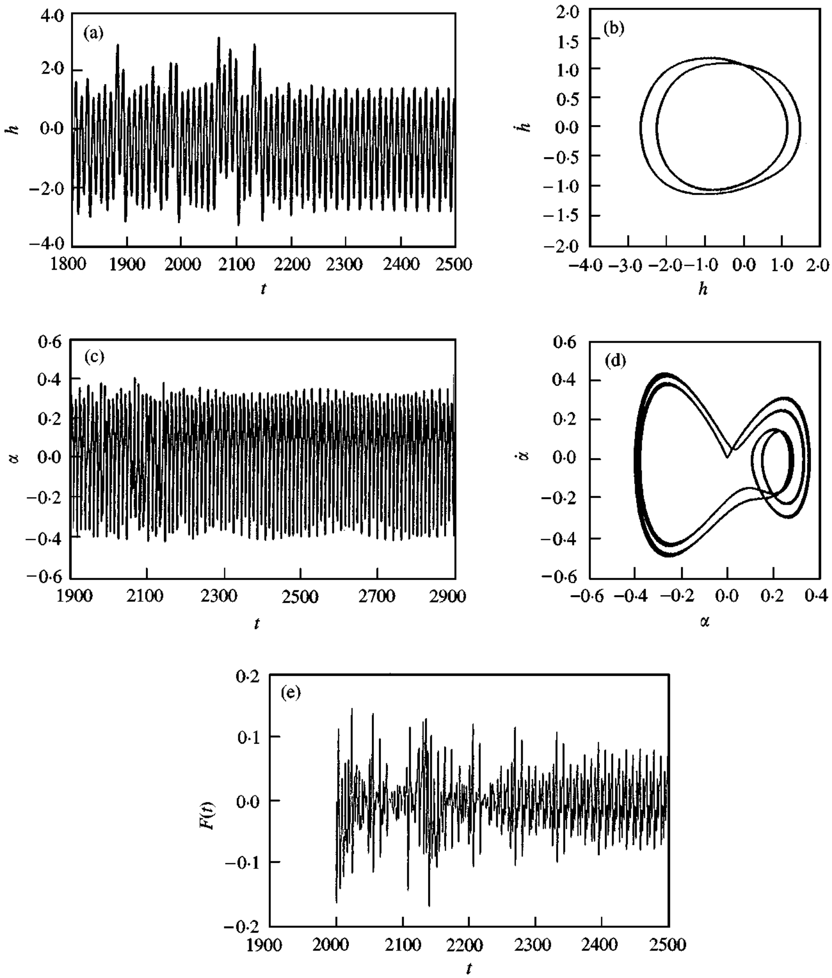

Figure 8. (a) Stabilization of plunging displacement ($K = 0{\cdot}3$, $\delta = 21{\cdot}5$); (b) corresponding phase plot of (a) after control transient (stabilization of period-2 orbit); (c) time history of pitching angle before and after control (stabilized to a period-2 cycle); (d) corresponding phase plot of (c); (e) control perturbations ($K = 0{\cdot}3$, $\delta = 21{\cdot}5$, control activated at $t = 2000$).

6. CONCLUSIONS

The chaotic motions of a two-dimensional airfoil with cubic pitching stiffness and linear viscous damping in incompressible flow have been controlled numerically by using Pyragas' method of delayed continuous feedback control. The control signal is used to perturb the airspeed parameter for effecting the control. Four cases have been analyzed by which the system's output is measured for input to the controller, namely, by measuring (i) plunging displacement, (ii) plunging velocity, (iii) pitching angle, and (iv) pitching velocity. It is observed that the system is stabilized to a periodic motion if the control is effected by either measuring the plunging displacement, or pitching angle, or pitching velocity. The control is not very effective for low values of control weight K if the control is applied by measuring the plunging velocity. Nevertheless, the control is achieved for this case at higher values of K, but at the cost of larger perturbations. For the (iii) and (iv) modes of control the control perturbations required are very small (less than 5% of the nominal value of the airspeed

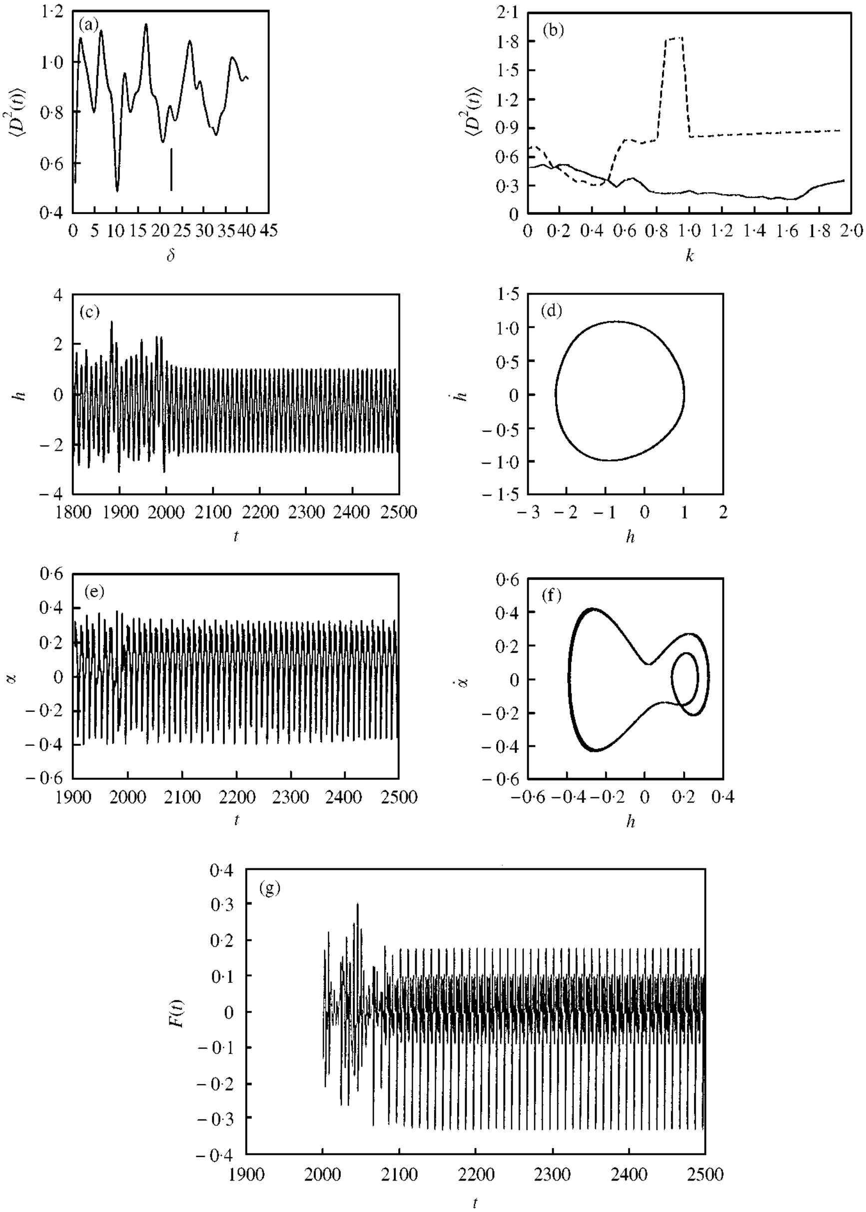

Figure 9. For case IV, when $x(t) = \dot{\alpha}(t)$: (a) dependence of the dispersion of perturbation on delay time; (b) dependence of the dispersion of the perturbation on K for two values of the delay time δ; ——, $\delta = 10{\cdot}75$; - - - -, $\delta = 21{\cdot}5$. (c) stabilization of plunging displacement ($K = 0{\cdot}4$, $\delta = 10{\cdot}75$); (d) corresponding phase plot of (c) after control transients (stabilization of period-1 orbit); (e) stabilization of pitching angle ($K = 0{\cdot}4$, $\delta = 10{\cdot}75$); (f) corresponding phase plot of (e) after control transients (stabilization of period-2 orbit); (g) control perturbations ($K = 0{\cdot}4$, $\delta = 10{\cdot}75$, control activated at $t = 2000$).

parameter). In (iii), the interval of K is very narrow for stabilization. The feedback control signal derived from the measurement of the pitching variables was found to be more effective in controlling the chaotic motion of the airfoil. From this analysis, it can be concluded that a higher-degree-of-freedom system can be controlled by using the delayed-feedback control method. For more effective control in all the modes (i)–(iv),

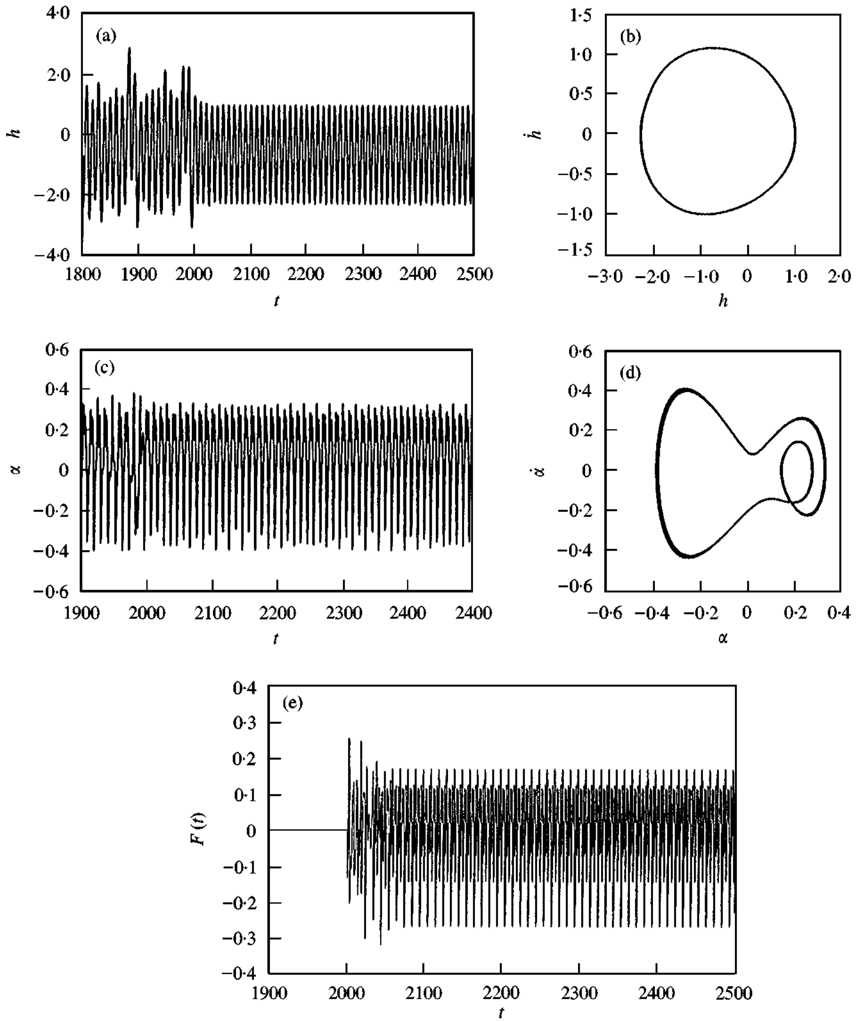

Figure 10. (a) Stabilization of plunging displacement ($K = 0{\cdot}4$, $\delta = 21{\cdot}5$); (b) corresponding phase plot of (a) after control transient (stabilization of period-1 orbit); (c) time history of pitching angle before and after control (stabilization of period-2 orbit); (d) corresponding phase plot of (c); (e) control perturbations ($K = 0{\cdot}4$, $\delta = 21{\cdot}5$, control activated at $t = 2000$).

a weighted sum of feedback control signal derived from plunging displacement, plunging velocity, pitching angle and pitching velocity could be attempted in future research.

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